DISTRIBUTION OF THE TRANSFER PARAMETERS DURING A REACTION BETWEEN THE MATERIAL OF A BODY SURFACE AND AN INJECTED SUBSTANCE WITH A BOUNDARY LAYER

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Described are the distributions of the Prandtl number, the Schmidt number, the Lewis number, the temperature, the enthalpy, the concentration, the diffusion currents, and the shear stresses across the height of a boundary layer with multicomponent diffusion.

When studying a reactive boundary layer, one considers either the reaction of the injected substance filtering through the porous wall with the gas in the outer stream, or the breakdown of the surface material of the body without injection [1].

The subsequent analysis here will take into account both these phenomena, i.e., the reaction of a carbon wall with hydrogen injected through it and an oxidizer in the outer laminar stream at infinite rates of both the heterogeneous and the homogeneous reaction:

$$2C + O_2 = 2CO, \tag{a}$$

$$2H_2 + O_2 = 2H_2O, \tag{b}$$

which occur within an infinitesimally thin zone at the wall surface.

The differential equations of transfer in a reacting transient boundary layer (during laminar flow) and the constraint equations have been transformed in [1] into the following self-adjoint form:

$$2\omega\omega'' + \tilde{\rho}\tilde{\mu}\tilde{u} = 0; \quad (\omega\bar{j}_i)' + \omega' C_i' = 0, \ i = 1,\ 2,\ 3;$$

$$\left\{\omega\left[-H'/\mathrm{Pr} - (1 - \mathrm{Pr}^{-1})\, u_\infty^2 \tilde{u} + \sum_i h_i (C_i/\mathrm{Pr} + \bar{j}_i)\right]\right\}' + \omega' H' = 0, \quad i = 1,\ 2,\ 3,\ 4. \tag{1}$$

$$\omega(1) = 0; \quad \omega'(0) = B^*/2; \quad C_1(1) = C_{1\infty};$$

$$C_i(1) = 0, \quad i = 2,\ 3; \quad 2r_{\mathrm{III}}\bar{J}_3 = \bar{J}_H;$$

$$2r_{\mathrm{II}}\bar{J}_2 = \bar{J}_C; \quad -2\bar{j}_1\omega(0) = r^{-1}\bar{J}_C + r_{\mathrm{I}}^{-1}\bar{J}_H;$$

$$2\omega'(0)\, Q(T_{\Pi}) = AT_e c_{P\Sigma e}\left[M(n) - M(1)\right] - 2\omega(0)\left(H' - \sum_i h_i C_i'\right)\mathrm{Pr}(0), \quad i = 1,\ 2,\ 3,\ 4, \tag{2}$$

where

$$\bar{j}_i = -C_i'/\mathrm{Sc}_i, \quad \bar{J}_i = \bar{j}_i\omega(0) + \omega'(0)\, C_i(0).$$

The prime sign following a symbol in (1)-(2) and in subsequent expressions denotes a derivative with respect to $\tilde{u}$ ($\tilde{u} = u/u_\infty$). The other designations are the same as in [1].

The composite system of nonlinear differential equations and constraints (1)-(2) describing the boundary layer at a porous wall with a chemical reaction between H_2, C, and oxygen O_2 of the outer stream as well as with a displacement of the wetted body surface due to reactions (a) and (b) will simplify appreciably

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when $Pr = Sc_i = 1$ and $\tilde{\mu}\tilde{\rho} = 1$. The solution for this case is given in [1].†

Problem (1)-(2) consisting of five second-order differential equations with 10 constraints, together with the system of N−1 relations between transforms of diffusion currents $\bar{j}_i$ ($\bar{j}_i = -C_i'/Sc_i$, i = 1, 2, 3) and derivatives C_i'

$$\frac{\mu}{\rho}\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33}\end{vmatrix}\begin{vmatrix}\bar{j}_1 \\ \bar{j}_2 \\ \bar{j}_3\end{vmatrix} = \begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33}\end{vmatrix}\begin{vmatrix} C_1' \\ C_2' \\ C_3'\end{vmatrix}, \tag{3}$$

obtained from expressions (33) in [1] with

$$\sum_i \bar{j}_i = 0, \quad \sum_i C_i = 1, \quad i = 1,\ 2,\ 3,\ 4, \tag{4}$$

where

$$a_{11} = C_1^{-1}\left[(1 - C_3)/m_4D_{14} + C_3/m_3D_{13}\right], \quad a_{12} = a_{32} = b_{12} = b_{32} = 0,$$
$$a_{13} = -(1/m_3D_{13} + 1/m_4D_{14}), \quad a_{21} = -(1/m_1D_{21} + 1/m_4D_{24}),$$
$$a_{31} = -(1/m_1D_{31} + 1/m_4D_{34}), \quad a_{33} = C_3^{-1}[C_1/m_1D_{31} + (1 - C_1)/m_4D_{34}],$$
$$b_{11} = -C_1^{-1}\left[C_3/m_3 + (1 - C_3)/m_4\right],$$
$$b_{13} = b_{23} = (1/m_3 - 1/m_4), \quad b_{21} = b_{31} = (1/m_1 - 1/m_4),$$
$$b_{22} = -C_2^{-1}\left[C_1/m_1 + C_3/m_3 + (1 - C_1 - C_3)/m_4\right],$$
$$b_{33} = C_3^{-1}\left[C_1/m_1 + (1 - C_1)/m_4\right],$$

and also together with the equations for determining the thermophysical properties of individual gas components (μ_i, λ_i, c_{pi}) as well as of their mixtures (u, λ, c_p) as functions of temperature T and concentration C_i‡) was solved by numerical integration on a Minsk-22 computer with the following step $S(\tilde{u})$ of the independent variable u: $S(\tilde{u}) = 0.05$ for $0 \le \tilde{u} \le 0.9$ and $S(\tilde{u}) = 0.03$ for $0.9 \le \tilde{u} \le 0.99$. The constraint problem (1)-(2) was reduced to a Cauchy problem by specifying the boundary conditions only at the wall surface at $\tilde{u} = 0$.

With quantities $C_{i\infty}$ (i = 1, 2, 3) and T_∞ given in (2) as well as with parameters u_∞ and B, the constraints for $\bar{j}_i(0)$ (i = 1, 2, 3), H(0), $\omega(0)$, and $\omega'(0)$ were found from relations (2), whereupon the constraint problem (1)-(2) was solved with (3) as well as the temperature-dependence and the concentration-dependence of the thermophysical properties taken into account. With parameters thus specified, the Cauchy problem becomes completely determinate. It is solved by the Runge–Kutta method. At the outer edge of the boundary layer, at the point $\tilde{u} = 1$, which is a singular point, the finite values of the sought parameters $\omega(1)$, $\bar{j}_1(1)$, $C_2(1)$, and $C_3(1)$ will be functions of the given approximations to the unknown quantities. For example,

$$\Phi\left[\omega(0),\ \bar{j}_1(0),\ C_2(0),\ C_3(0)\right] = |\Delta\omega(1)|^2 + |\Delta\bar{j}_1(1)|^2 + |\Delta C_2(1)|^2 + |\Delta C_3(1)|^2,$$

where Δ denotes the difference between following and preceding values of respective parameters at point $\tilde{u} = 1$. When $\Phi \to 0$, the unknown quantities ω, j_i, and C_i become determinate. (The minimum of function Φ is determined by the gradient method [3].) The constraints (2) at $\tilde{u} = 1$ will also be satisfied within the appropriate accuracy, i.e., the solution to the Cauchy problem with initial values determined in this manner does, evidently, coincide with the solution to the original constraint problem. The results of numerical integration have been tabulated for various values of the injection number B, flow velocities u_∞, and temperatures T_∞, with the variation of thermophysical properties across the boundary layer taken into account (Figs. 1-4, Tables 1-2). Numerical results have also been obtained for $Pr = Sc_i = 1$ and $\mu\rho$ = const (the curves in Figs. 1-3 and the values in Tables 1-2 correspond to u_∞ = 15 m/sec).

According to the curves in Figs. 1-2, the normalized diffusion currents $\bar{j}_i$ as well as the Schmidt numbers Sc_i and the Lewis numbers Le_i vary appreciably within the $0.8 \le \tilde{u} \le 1$ zone adjacent to the outer edge of the boundary layer. Within the $\tilde{u} < 0.8$ zone all these quantities vary negligibly little. The largest variation across the boundary layer is characteristic of the numbers Le_i and Sc_i. Thus, while $Le_i = 0.835$,

† In ([1], p. 66) $u_0^{-1/2} = B^*/2$ should be corrected to $\tan\gamma u_0^{-1/2} = B^*/2$.

‡ This is relation (34) in [1].

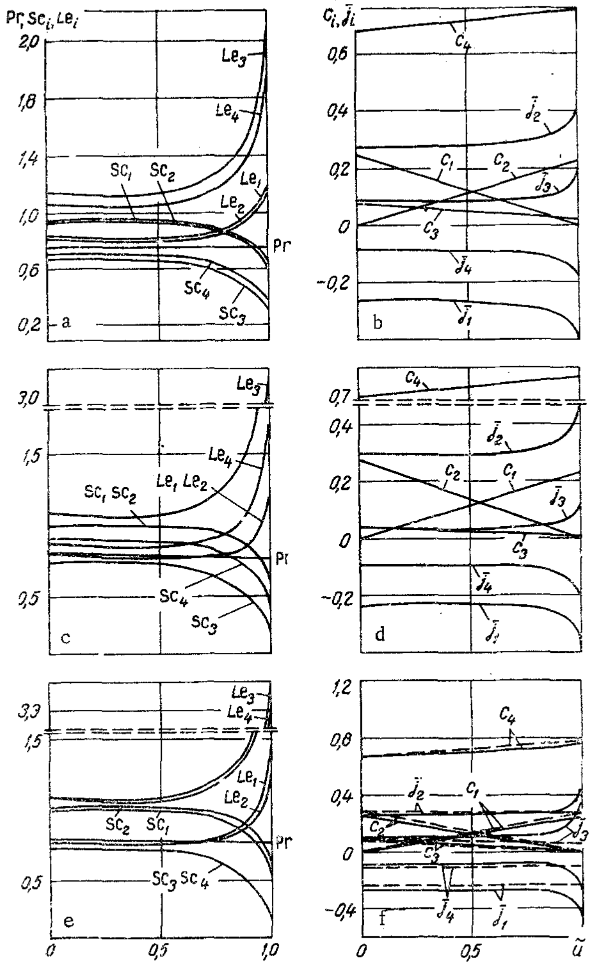

Fig. 1

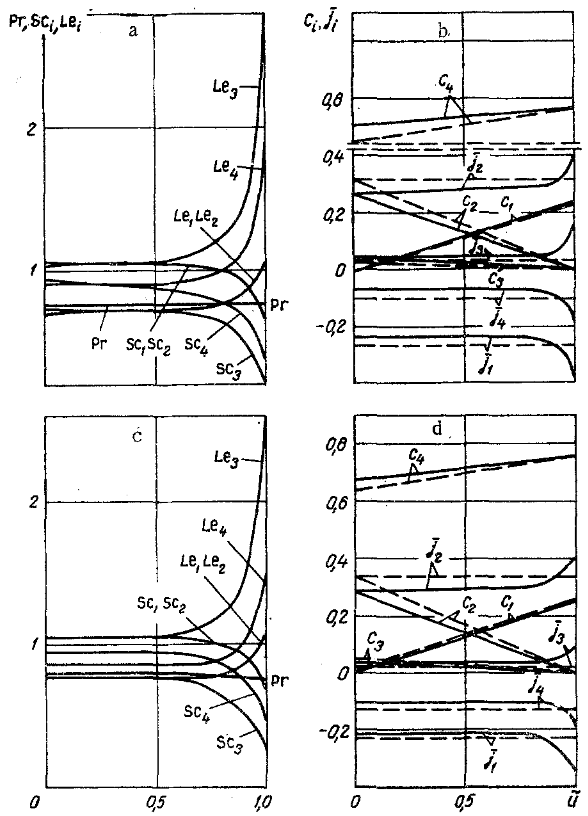

Fig. 2

Fig. 1. Variation of numbers Pr, Sc_i, Le_i, and of C_i, $\bar{j}_i$ across the height $\tilde{u}$ of a boundary layer, at various surface temperatures T_s of the porous wall and at various values of the injection number B: a, b) $T_s = 1300°K$, $P = 0.2$, $B = 75$; c, d) $T_s = 2000°K$, $P = 0.2$, $B = 30$; e, f) $T_s = 2000°K$, $P = 0.2$, $B = 78.7$; in f) the solid lines correspond to $Pr \neq Sc_i \neq 1$ and the dashed lines correspond to $Pr = Sc_i = 1$.

Fig. 2. Numbers Pr, Sc_i, Le_i, and C_i, $\bar{j}_i$ as functions of $\tilde{u}$, at various porosities P: a, b) $T_s = 2500°K$, $P = 0.4$, $B = 25$; c, d) $T_s = 2500°K$, $P = 0.2$, $B = 25$; in b) the solid lines correspond to $Pr \neq Sc_i \neq 1$ and the dashed lines correspond to $Pr = Sc_i = 1$.

$Le_2 = 0.826$, $Le_3 = 1.136$, and $Le_4 = 1.058$ at $\tilde{u} = 0$ (Fig. 1a), for example, $Le_1 = 1.175$, $Le_2 = 1.139$, $Le_3 = 2.261$, and $Le_4 = 1.963$ at $\tilde{u} = 0.99 \approx 1$. The numbers Sc_3 and Sc_4 are at $\tilde{u} = 0$ approximately twice as high as at $\tilde{u} = 1$, the ratios are $Sc_i(0)/Sc_i(1) \approx 1.45$ and $\bar{j}_i(0)/\bar{j}_i(1) \approx 0.7$ for $i = 1, 2$ (Fig. 1b) but $\bar{j}_i(0)/\bar{j}_i(1) \approx 0.5$ for $i = 3, 4$. It is to be noted that the Prandtl number (Fig. 1a) varies only slightly (and almost linearly) across the boundary layer (thus, $Pr = 0.783$ and $Pr = 0.756$ at $\tilde{u} = 0$ and $\tilde{u} = 1$, respectively). Similar variations in parameters Pr, $\bar{j}_i$, Sc_i, and Le_i ($i = 1, 2, 3, 4$) are noted also in other cases (Figs. 1-2), with the relation $C_i = C_i(\tilde{u})$, $0 \leq \tilde{u} \leq 1$ for the concentration being almost linear. The absolute values of $\bar{j}_i$ and C_i are very different when $Pr \neq Sc_i \neq 1$ and when $Pr = Sc_i = 1$. According to Figs. 1-2, $Pr = Sc_i = 1$ does not correspond to actual physical conditions. In many cases one may assume, however, that Pr = const across the boundary layer. As the injection of hydrogen (the B number) through the porous wall is increased, $C_3(0)$ and $\bar{j}_3$ increase inasmuch as the entire injected hydrogen H_2 converts into water H_2O according to reaction (b), while $C_2(0)$ and $\bar{j}_2$ decrease, but $C_4(0)$ and $\bar{j}_4$ change insignificantly inasmuch as nitrogen N_2 is in this case an inert substance. The concentrations $C_1(\tilde{u})$ and the normalized currents $\bar{j}_1(\tilde{u})$ remain invariable as the injection number B is increased (Figs. 1-2), because the entire oxygen O_2 (at a given concentration $C_{1\infty}$) is taken up stoichiometrically in reactions (a) and (b). Numbers Pr and Sc_i ($i = 1, 2, 3, 4$), which are functions of the thermophysical properties u, $\bar{c}_p$, λ, ρ, and D_i determined from the composite system of relations

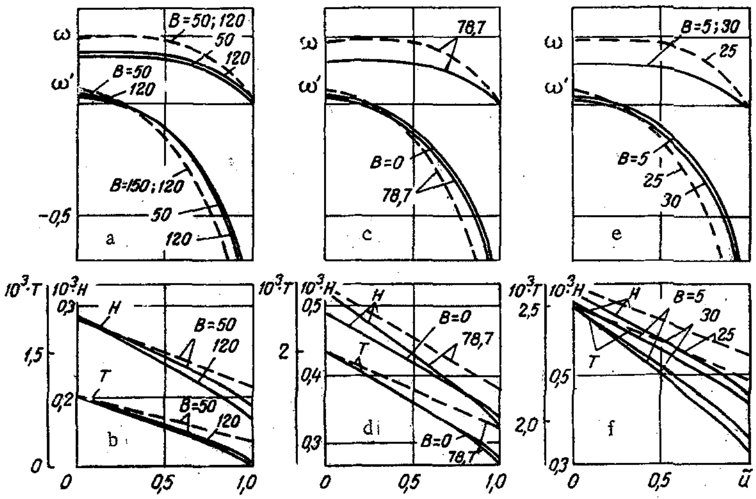

Fig. 3. Parameters ω and ω', enthalpy H, and temperature T as functions of $\tilde{u}$, at various surface temperatures T_S and various values of the injection number B: a, b) P = 0.2, T_S = 1300°K; c, d) P = 0.2 and T_S = 2000°K; e, f) P = 0.2 and T_S = 2500°K. The solid lines correspond to Pr ≠ 1 and the dashed lines correspond to Pr = Sc_i = 1.

TABLE 1. Variation of Parameters ω, ω', H (kcal/kg), and T (°K) across the Height of a Boundary Layer, at P = 0.4

$\tilde{u}$	$Pr \neq Sc_i \neq 1$				$Pr = Sc_i = 1$			
	$\omega \cdot 10^3$	$\omega' \cdot 10^3$	H	T	$\omega \cdot 10^3$	$\omega' \cdot 10^3$	H	T
0,00	186	44	685	2500	250	118	705	2500
0,10	190	39	657	2431	261	108		2462
0,20	193	22	628	2361	271	80		2422
0,30	194	— 5	600	2290	276	34		2382
0,40	192	— 43	572	2218	277	— 29		2341
0,50	185	— 94	544	2145	270	— 111	607	2298
0,60	172	— 159	516	2069	254	— 216		2255
0,70	152	— 245	487	1991	226	— 351		2210
0,80	122	— 362	458	1908	182	— 535		2165
0,90	78	— 548	426	1817	116	— 822		2118
0,96	39	— 763	405	1755	58	—2251		2089
0,99	13	—1029	392	1716	18	—1568		2075
1,00	0	—∞	390	1702	0	—∞		2070

(33) and (34) in [1], vary in our cases (Figs. 1-2) as follows as the B number is increased: the Pr number remains almost unchanged, the Sc_i (i = 1, 2, 4) numbers decrease, the Sc_3 number increases slightly. Considering that $Pr(\tilde{u}) \approx$ const within $0 \le \tilde{u} \le 1$, an increase or decrease in an Sc_i number causes the Le_i number ($Le_i = Pr/Sc_i$) to change in the opposite sense. It is to be noted that in the vicinity of point $\tilde{u} = 1$, which is singular, the error of numerical integration is maximum and, consequently, the largest deviation of calculated from exact values of parameters is possible here.

A comparison of Fig. 1a, b with Fig. 1e, f and Fig. 1c, d with Fig. 2c, d shows that, as the surface temperature T_S rises, the $Sc_i(0)$ numbers increase while the $Sc_i(1)$ numbers decrease. The concentrations C_i are almost the same but the normalized currents $\bar{j}_i$ differ appreciably at both surface temperatures as $\tilde{u} \to 1$. When the porosity is increased from 0.2 (Fig. 2c, d) to 0.4 (Fig. 2a, b), Sc_3 and Sc_4 decrease (Le_3 and Le_4 increase correspondingly). All other parameters remain approximately the same.

Variations of ω ($\omega = \tilde{\mu} d\tilde{u}/d\eta$) and ω' ($\omega' = d\omega/d\tilde{u}$) as well as of enthalpy H (kcal/kg) and temperature (°K) across the boundary layer are shown in Fig. 3. In Table 1 these parameters are listed for P = 0.4, B = 30, and T_S = 2500°K. For the absolute values of B, T_S, and P considered here, a change of B, rather than of T_S or P, causes the most appreciable change in ω and ω'. Since ω and ω' are functions of the $\tilde{\mu}\tilde{\rho}$ factor in the first of Eqs. (1), hence we have for P = 0.2 (Fig. 3c) or for P = 0.4 (Table 1) with B = 30:

$$\omega'|_{P=0.2} < \omega'|_{P=0.4}, \quad \omega|_{P=0.2} > \omega|_{P=0.4}.$$

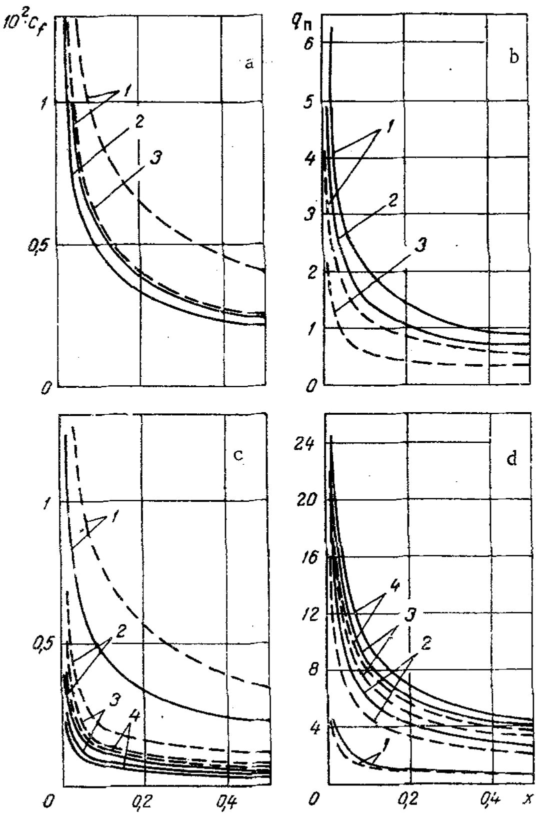

Fig. 4. Friction coefficient c_f and thermal flux q_s as functions of the x-coordinate along a porous wall (P = 0.2), at various surface temperature T_s and various stream velocities u_∞: a, b) B = 25, u_∞ = 15 m/sec. 1) T_s = 2500°K; 2) 2000; 3) 1300; c, d) B = 78.7, T_s = 2000°K, u_∞ = 15 m/sec. 1) T_s = 2500°K; 2) 2000; 3) 1300; c, d) B = 78.7, T_s = 2000°K; 1) u_∞ = 15 m/sec; 2) 200; 3) 400; 4) 500. The solid lines correspond to $Pr \neq Sc_i \neq 1$ and the dashed lines correspond to $Pr = Sc_i = 1$. Distance x (m).

TABLE 2. Variation of the Friction Coefficient c_f and the Thermal Flux q_s (kcal/m^2 · sec) along the Porous Wall (x-Coordinate), at T_s = 2000°K, u_∞ = 15 m/sec, B = 78.7

x, cm		1	3	6	10	15	20	30	50
				$Pr \neq Sc_i \neq 1$					
$c_f \cdot 10^5$	P=0,2	1533	885	626	485	396	343	280	217
	P=0,3	1496	864	611	473	386	335	273	212
	P=0,35	1475	852	602	466	381	330	269	209
$q_s \cdot 10^3$	P=0,2	5131	2962	2095	1622	1325	1147	937	726
	P=0,3	5609	3238	2290	1774	1449	1254	1024	793
	P=0,35	5853	3379	2390	1851	1511	1309	1069	828
				$Pr = Sc_i = 1$					
$c_f \cdot 10^5$	P=0,2	2489	1437	1016	787	643	557	454	352
	P=0,3	2426	1400	990	767	626	542	443	343
$q_s \cdot 10^3$	P=0,2	4318	2493	1763	1365	1115	965	788	611
	P=0,3	4785	2763	1954	1513	1236	1070	847	677

Furthermore, if H(0) and T(0) are, respectively, the same at P = 0.2 and at P = 0.4, then

$$T(1)|_{P=0.2} > T(1)|_{P=0.4}, \quad H(1)|_{P=0.2} > H(1)|_{P=0.4}, \tag{5}$$

because, as the porosity of the graphite wall decreases, both the hydrogen current component $\bar{j}_H$ and T(1) (or H(1)) increase, inasmuch as the heat of the exothermal reaction (b) is higher than that of reaction (a); in other words, the given temperatures T_s (T_s = T(0)) is reached at a lower ambient temperature T_∞ (T_∞ = T(1)) when P = 0.4 than when P = 0.2. The increase in ω and ω' when $Pr = Sc_i = \widetilde{\mu\rho} = 1$, as compared to their increase when $Pr \neq Sc_i \neq 1$, is related to the specifics of the first equation in system (1), where the largest values of ω and ω' correspond to the conditions $\widetilde{\mu\rho} \to 1$ and $Pr = Sc_i = 1$ (Fig. 3, Table 1).

As temperature T_s rises (Fig. 4a, b), the local values of the friction coefficient c_f and of the thermal flux q_s defined as

$$c_f = \frac{2\omega(0)}{\sqrt{\mathrm{Re}_x}}, \quad q_s = \left[\frac{\lambda(0)}{\mu(0)}\omega(0)\frac{\rho_\infty u_\infty}{\sqrt{\mathrm{Re}_x}}\right] T'(0) \tag{6}$$

increase, with $\omega(0) = u_0^{-3/2}$ when $\mathrm{Pr} = \mathrm{Sc}_i = 1$. Such an increase in c_f and q_s is due to a net change not only in the thermophysical properties in (6) but also in $T'(0)$ and $\omega(0)$. The decrease in c_f and the increase in q_s with increasing u_∞ (Fig. 4c, d) is explained in the light of relations (6), where $c_f \sim u_\infty^{-1/2}$ and $q_s \sim u_\infty^{1/2}$. A higher porosity of the graphite wall (Table 2), too, will reduce c_f and raise q_s because, as has been mentioned earlier, a higher P allows the temperature T_∞ necessary for reaching the given temperature T_s to be lower and thus makes for a higher Reynolds number Re_x but a lower $\omega(0)$ and thus for a lower c_f and a higher q_s, the latter depending mainly on the temperature difference $\Delta T = T_s - T_\infty$ ($q_s \sim \Delta T$).

According to Fig. 4 and Table 2, c_f is higher and q_s is lower when $\mathrm{Pr} = \mathrm{Sc}_i = 1$ than when $\mathrm{Pr} \neq \mathrm{Sc}_i \neq 1$. The inequality

$$c_f|_{\mathrm{Pr}=\mathrm{Sc}_i=1} > c_f|_{\mathrm{Pr}\neq\mathrm{Sc}_i\neq 1}$$

is in the former case explained by an increase in $\omega(0)$ and a decrease in Re_x, but in the latter case q_s increases because the derivative $T'(0)$ ($T' = dT/d\tilde{u}$) is larger regardless of the decrease in $\omega(0)$ and the increase in Re_x in (6). Thus, in the first case and in the second case the factor inside the square brackets and $T'(0)$ in (6) are equal to $133 \cdot 10^{-5}$, 532 and $119 \cdot 10^{-5}$, 290, respectively ($T_s = 2500°$K, B = 25, P = 0.2, x = 1 mm, $u_\infty = 15$ m/sec). The stream temperature T_∞ is 2180°K for the first case and 1910°K for the second case. In this way, according to Figs. 1-4 and Tables 1-2, the assumption $\mathrm{Pr} = \mathrm{Sc}_i = 1$ leads in our case to large deviations values from exact values.

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